2.2 – Evaluating Determinants by Row Reduction

Theorem 2.2.1 Let A be a square matrix. If A has a row of zeros or a column of zeros, then $\det(A) = 0$.

Theorem 2.2.2 Let A be a square matrix. Then $\det(A) = \det(A^T)$.

Theorem 2.2.3 Elementary Row Operations

Let A be an $n \times n$ matrix.

a) If B is the matrix that results when a single row or single column of A is multiplied by a scalar k, then $\det(B) = k\det(A)$.
b) If B is the matrix that results when two rows or two columns of A are interchanged, then $\det(B) = -\det(A)$.
c) If B is the matrix that results when a multiple of one row of A is added to another or when a multiple of one column is added to another, then $\det(B) = \det(A)$.

Theorem 2.2.4 Let E be an $n \times n$ elementary matrix.

a) If E results from multiplying a single row of I_n by a nonzero number k, then $\det(E) = k$.

b) If E results from interchanging two rows of I_n, then $\det(E) = -1$.

c) If E results from adding a multiple of one row of I_n to another, then $\det(E) = 1$.

12. Evaluate the determinant of the matrix by first reducing the matrix to row echelon form and then using some combination of row operations and cofactor expansion.

$$\begin{bmatrix} 1 & -3 & 0 \\ -2 & 4 & 1 \\ 5 & -2 & 2 \end{bmatrix}$$

In #16 and 20, evaluate the determinant, given that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -6.$$

16. $\begin{vmatrix} g & h & i \\ d & e & f \\ a & b & c \end{vmatrix}$

20. $\begin{vmatrix} a & b & c \\ 2d & 2e & 2f \\ g+3a & h+3b & i+3c \end{vmatrix}$

Theorem 2.2.5 If A is a square matrix with two proportional rows or two proportional columns, then $\det(A) = 0$.